A PROJECT

ON

APPLICATIONS OF DERIVATIVE

SUBMIITED IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF DEGREE OF

BACHELOR OF SCIENCE

IN

MATHEMATICS

SUBMITTED BY

1. NAJNEEN YASEEN KACHHI
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5. AMIT LAKHANLAL GUPTA
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7. ASAL BANDUJI PENDAM

UNDER THE GUIDANCE OF

DR. RAVIKANT MISHRA
Assistant Professor

DEPARTMENT OF MATHEMATICS

BHIWAPUR MAHAVIDYALAYA, BHIWAPUR

2022-2023

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DECLARATION

This Project work entitled **"APPLICATIONS OF DERIVATIVE"** is our own work carried out under the guidance of **Asst. Prof. Dr. Ravikant Mishra,** Department of Mathematics, Bhiwapur Mahavidyalaya, Bhiwapur, Nagpur. This work in the same form or in any other form is not submitted by me or by anyone else for the award of any degree.

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CERTIFICATE

This is to certify that the Project work entitled **"Applications of Derivative"** , is the bonafide work done by student and is submitted to BhiwapurMahavidyalaya, Bhiwapur, Dist-Nagpur for the partial fulfillment of the requirements for the degree of Bachelor of Science in Subject name Mathematics

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"Applications of Derivative"

Introduction

1.1 History

Newton and Leibniz quite Independently of one another were largely responsible for developing the ideas of Integral calculus to the point where hitherto Insurmountable problems could be solved by more or less routine methods. The successful accomplishments of these men were primarily due to the fact that they were able to fuse together the integral calculus with the second main branch of calculus, differential calculus.

Isaac Newton (1642-1727)

Gottfried Leibniz (1646-1716)

The central idea of differential calculus is the notation of derivative. Like the integral, the derivative originated from a problem in geometry the problem finding the tangent line at a point of a curve. Unlike the integral. However, the derivative evolved very late in the history of mathematics. The concept was not formulated until early in the 17 century when the French mathematician Pierre de Fermat, attempted to determine the maxima and minima of certain special functions.

1.2 Definition of Derivative:

We begin with a function f defined at least on some open interval (a, b) on the x-axis. Then we choose a fixed-point x in this interval and introduce the difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

Where the number h, which may be positive or negative (but not zero), is such that $x + h$ also lies in (a, b). The numerator of this quotient measures the change in the function when x changes from x to x + h. The quotient itself is referred to as the average rate of change of f in the interval joining x to x + h.

Now we let h approach zero and see what happens to this quotient. If the quotient approaches some definite value as a limit (which implies that the limit is the same whether h approaches zero through positive values or through negative values), then this limit is called the derivative of f at x and is denoted by the symbol f'(x) (read as "f prime of x"). Thus, the formal definition of f(x) may be stated as follows:

2. DEFINITION OF DERIVATIVE: The derivative f'(x) is defined by the equation

$$F'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. The number f'(x) is also called the rate of change of f at X.

Meaning of derivative:-

- The Derivative is the exact rate at which one quantity changes with respect to another
- Geometrically, the derivative is the slope of curve at the point on the curve. o The derivative is often called the "instantaneous" rate of change.
- The derivative of a function represents an infinitely small change
- the function with respect to one of its variables.
- The Process of finding the derivative is called "differentiation".

1.3 Application of Derivatives in Various Fields/Science such as in: -

- Biology
- Economics
- Chemistry
- Physics
- Mathematics
- Others (Psychology, sociology & geology)

2. Application of Derivative in Medical and Biology:

Sometimes we may question ourselves why students in biology or medical department still have to take mathematics and even physics. After reading this post, you will understand why.

2.1 Growth Rate of Tumor:

A tumor is an abnormal growth of cells that serves no purpose. There are certain levels of a tumor regarding to its malignancy.

The first level is benign tumor. It does not invade nearby tissue or spread to other parts of the body the way cancer can. In most cases, the outlook with benign tumors is very good. But benign tumors can be serious if they press on vital structures such as blood vessels or nerves. Therefore, sometimes they require treatment and other times they do not.

The second level is premalignant or precancerous tumor which is not yet malignant, but is about to become so.

The last level is malignant tumors. These are cancerous tumors; they tend to become progressively worse, and can potentially result in death. Unlike benign tumors, malignant ones grow fast, they are ambitious, they seek out new territory, and they spread (metastasize).

The abnormal cells that form a malignant tumor multiply at a faster rate. Experts say that there is no clear dividing line between cancerous, precancerous and non-cancerous tumors sometimes determining which is which may be arbitrary, especially if the tumor is in the middle of the spectrum. Some benign tumors eventually become premalignant, and then malignant.

The rate at which a tumor grows is directly proportional to its volume. Larger tumors grow faster and smaller tumors grow slower.

The volume of a tumor is found by using the exponential growth model which is

$$V(t) = V_0 * e^{kt}$$

Vo initial volume exponential growth, k-growth constant, t-time

In order to find the rate of change in tumor growth, you must take the derivative of the volume equation (V (t))

$$V(t) = V_0 * e^{kt}$$

$$V'(t) = V_0 * e^{kt} * k$$

Because e^{kt} is a complicated function, we use chain rule to derivate it.

$$y = e^{kt}$$

Let $u = kty = e^u$

$$\frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt}\frac{dy}{dt} = e^u$$

$$\frac{du}{dt} = k\frac{dy}{dt} = ke^u$$

$$\frac{dy}{dt} = ke^{kt}$$

From the calculation above, we know that the derivative of e^{kt} is $k * e^{kt}$

$$V'(t) = V_0 * k * e^{kt}$$

Because V(t) itself is equal to $V_0 * e^{kt}$ we may conclude

$$V'(t) = k * V$$

There is the example to prove this theory:

2.2 Larger tumor:

Find the rate of change of a tumor when its initial volume is 10 cm^3 with a growth constant of 0.075 over a time period of 7 years

$$V(t) = V_0 * e^{kt}$$

$$V(7) = 10 \times 2.178^{(0.075)7}$$

$$V(7) = 15.05cm^3$$

$$V'(t) = k.v$$

$$V'(t) = 0.075 \times 15.05$$

$$V'(t) = 1.13cm^3/year$$

Then let's calculate the rate of change of smaller tumor with the same growth constant and time period.

2.3 Smaller tumor:

Find the rate of change of a tumor when its initial volume is 2 cm^3 with a growth constant of 0.075 over a time period of 7 years

$$V(t) = V_0 * e^{kt}$$

$$V(7) = 2 \times 2.178^{(0.075)7}$$

$$V(7) = 3.01cm^3$$

$$V'(t) = k.v$$

$$V'(t) = 0.075 \times 3.01$$

$$V'(t) = 0.23cm^3/year$$

With this calculation we know how important it is to detect a tumor as soon as possible. It is crucial to give a right treatment that will stop or slow down the growth of the tumor because bigger tumor intend to grow faster and, in some case, becoming a cancer that have a small chance to cured.

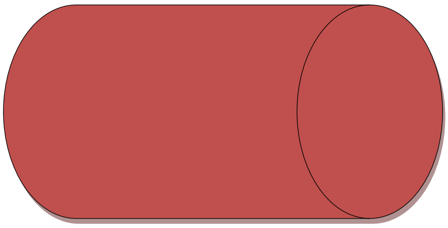
2.4 Blood Flow:

High blood pressure can affect the ability of the arteries to open and close. If your blood pressure is too high, the muscles in the artery wall will respond by pushing back harder. This will make them grow bigger, which makes your artery walls thicker. Thicker arteries mean that there is less space for the blood to flow through. This will raise your blood pressure even further.

Due to fat and cholesterol plaque that cling to the vessel, it becomes constricted. If an artery bursts or becomes blocked, the part of the body that gets its blood from that artery will be starved of the energy and oxygen it needs and the cells in the affected area will die.

If the burst artery supplies a part of the brain then the result is a stroke. If the burst artery supplies a part of the heart, then that area of heart muscle will die, causing a heart attack.

We can calculate the velocity of the blood flow and detect if there are something wrong with the blood pressure or the blood vessel wall. In this case, we portrait the blood vessel as a cylindrical tube with radius R and length L as illustrated below



Because of the friction at the walls of the vessel, the velocity of the blood is not the same in every point. The velocity of the blood in the center of the vessel is faster than the flow of the blood near the wall of the vessel. The velocity is decreases as the distance of radius from the axis (center of the vessel) increases until v become 0 at the wall.

The relationship between velocity and radius is given by the law of laminar flow discovered by the France Physician Jean-Louis-Marie Poiseuille in 1840. This state that

$$V = \frac{P}{4nL}(R^2 - r^2)$$

V = initial volume

n = viscosity of the blood

P = Pressure difference between the ends of the blood vessel

L = length of the blood vessel

R = radius of the blood vessel

r = radius of the specific point inside the blood vessel that we want to know.

To calculate the velocity gradient or the rate of change of the specific point in the blood vessel we derivate the law of laminar flaw

$$V = \frac{P}{4nL}(R^2 - r^2)$$

$$V' = \frac{d}{dr}\left[\frac{P}{4nL}(R^2 - r^2)\right] = \frac{P}{4nL}\frac{d}{dt}(R^2 - r^2)$$

$$V' = \frac{P}{4nL}(0 - 2r)$$

$$V' = \frac{P}{4nL}(-2r)$$

Example: - The left radial artery radius is approximately 2.2 mm and the viscosity of the blood is 0.0027 Ns/m^2. The length of this vessel is 20 mm and pressure differences are 0.05 N. What is the velocity gradient at r = 1 mm from center of the vessel?

$$V' = \frac{2rP}{4nL}$$

$$V' = \frac{-2.1 \times 10^{-3} \times 0.05}{4 \times 0.0027 \times 20 \times 10^{-3}}$$

$$V' = \frac{-10^{-4}}{2.16 \times 10^{-4}}$$

$$V' = -0.46\, m/s$$

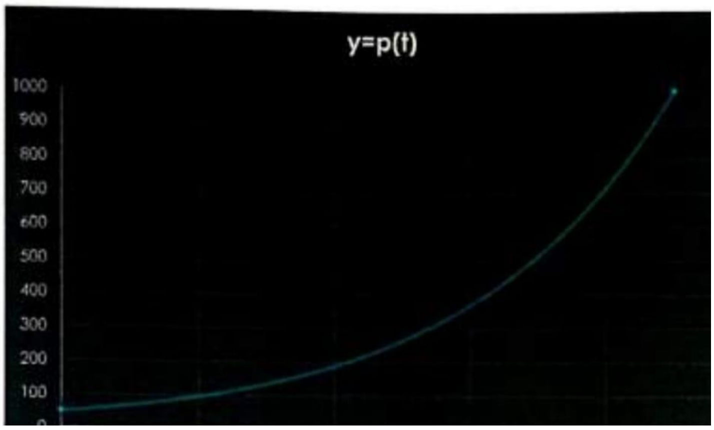
So, we can conclude that the velocity gradient is -0.46 m/s. if the gradient of velocity is too high then the person may have a constriction in his/her blood vessel and needs further examination and treatment.

2.5 Population models:

The population of a colony of plants, or animals, or bacteria, or humans, is often described by an equation involving a rate of change (this is called a "differential equation"). For instance, if there is plenty of food and there are no predators, the population will grow in proportion to how many are already there:

$$\frac{dp}{dt} = rp$$

Where r is constant. It's not hard to check that the function p(t) $= P_0 e^{rt}$



$P_0 = 50$, $r = 0.65$

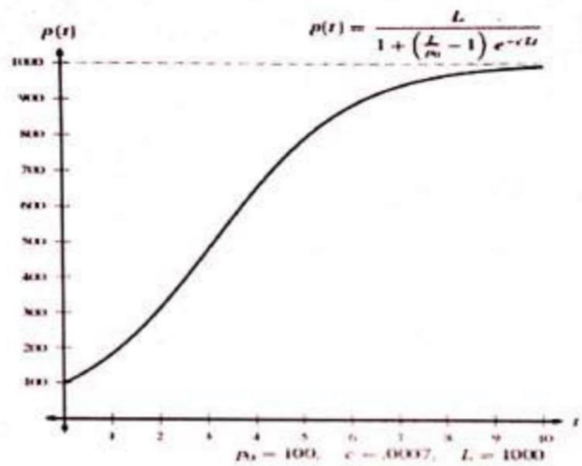
Satisfies this differential equation, where po is the starting population. Colonies tend to grow exponential until they run out of space food or run into predators.

When there are limits on the food supply, the population is often governed by the logistic

EQUATION:-

$$\frac{dp}{dt} = cp(L - p)$$

Where c and L are constant. The population grows exponentially for a while, and then levels off at a horizontal asymptote of L

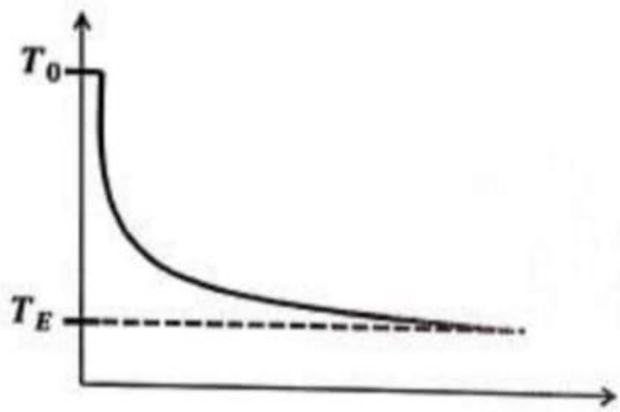


The logistic equation also governs the growth of epidemics, as well as for the example, the frequency of certain genes in a population.

3. Application of Derivative in Chemistry

The change in temperature

- An object's temperature over time will approach the temperature of its surroundings (the medium).
- The greater the difference between the object's temperature and the medium's temperature, the greater the rate of change of the object's temperature.
- This change is a form of exponential decay.



3.1 Newton's Law of Cooling.

- It is a direct application for differential equations.
- Formulated by Sir Isaac Newton.
- Have many applications in our everyday life.
- Sir Isaac Newton found this equation behaves like what is called in Math (differential equations) so his used some techniques to find its general solution.

3.2 Derivation of Newton's Law of Cooling:

- **Newton's observations:**

He observed that observed that the temperature of the body is proportional to the difference between its own temperature and the temperature of the objects in contact with it.

- Formulating: First order separable DE
- Applying differential calculus:

$$\frac{dT}{dt} = -k(T - T_g)$$

Where k is the positive proportionality constant

- By separation of variables we get

$$\frac{dT}{(T - T_g)} = -kdt$$

- By integrating both sides we get

$$In(T - T_g) + C - kt$$

- At time $(t = 0)$ the temperature is T_0

$$C = -In(T_0 - T_g)$$

- By substituting $C = -In(T_0 - T_g)$we get

$$In\ \frac{(T - T_E)}{(T_0 - T_E)} = -kt$$

$$T = T_E + (T_0 - T_E)e^{-kt}$$

3.3 Applications of Newton's Law of Cooling in Investigations in a Crime Scene:

The police came to a house at 10:23 am were a murder had taken place. The detective measured the temperature of the victim's body and found that it was 26.7°C. Then he used a thermostat to measure the temperature of the room that was found to be 20°C through the last three days. After an hour he measured the temperature of the body again and found that the temperature was 25.8°C. Assuming that the body temperature was normal (37°C), what is the time of death?

Solution $T = T_E + (T_0 - T_E)e^{-kt}$

Let the time at which the death took place be x hours before the arrival of the police men.

Substitute by the given values

$$T(x) = 26.7 = 20 + (37 - 20)e^{-kx}$$

$$T(x+1) = 25.8 = 20 + (37 - 20)e^{-k(x+1)}$$

Solve the 2 equations simultaneously $\quad 0.394 = e^{-kx}$

$$0.341 = e^{-k(x+1)}$$

By taking the logarithmic function

$In(0.394) = -kx$.........(1) $\qquad In(0.341) = -k(x+1)$............(2)

By dividing (1) by (2)

$$\frac{In(0.394)}{In(0.341)} = \frac{-kx}{-k(x+1)}$$

$$0.8657 = \frac{x}{x+1}$$

Thus, $x \cong$ 7 hours

Therefore, the murder took place 7 hours before the arrival of the detective which is at 3:23 pm

3.4 Applications of Newton's Law of Cooling in Processor Manufacturing

A global company such as Intel is willing to produce a new cooling system for their processors that can cool the processors from a temperature of 50°C to 27°C in just half an hour when the temperature outside is 20°C but they don't know what kind of materials they should use or what the surface area and the geometry of the shape are. So, what should they do?

Simply they have to use the general formula of Newton's law of cooling

$$T = T_E + (T_0 - T_E)e^{-kt}$$

And by substituting the numbers they get$27 = 20 + (50 - 20)e^{-0.5k}$

Solving for k, we get $\quad K = 2.9$

so, they need a material with K=2.9 (k is a constant that is related to the heat capacity, thermodynamics of the material and also the shape and the geometry of the material)

Application of Derivative in Physics: Derivatives with respect to time:

In physics, we are often looking at how things change over time:

1. Velocity is the derivative of position with respect to time:

$$v(t) = \frac{d}{dt}(x(t))$$

2. Acceleration is the derivative of velocity with respect to time:

$$a(t) = \frac{d}{dt}(v(t) = \frac{d^2}{dt^2}(x(t))$$

3. Momentum (usually denoted p) is mass times velocity, and force (F) is mass times acceleration, so the derivative of momentum is

$$\frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma = F$$

One of Newton's laws says that for every action there is an equal and opposite reaction, meaning that if particle 2 puts force F on particle 1, then particle 1 must put force-F on particle 2. But this means that the (momentum is constant), since

$$\frac{d}{dt}(p_1 + p_2) = \frac{dp_1}{dt} + \frac{dp_2}{dt} = F - F = 0$$

This is the law of conservation of momentum.

Derivatives with Respect to Position:

In physics, we also take derivatives with respect to x.

1. For so called "conservative" forces, there is a function $V(x)$ such that the force depends only on position and is minus the derivative of V namally $F(x) = \frac{dv(x)}{dx}$ $(The\ function\ v(x)$ is called the potential energy. dx For instance, for a mass on a spring the **potential energy** is $\frac{1}{2}kx^2$, where k is a constant and the force is -k x.

2. The **kinetic energy** is $\frac{1}{2}mv^2$. Using the chain rule, we find that the **total energy** is

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 + v(x)\right) = mv\frac{dv}{dt} + V'(x)\frac{dx}{dt} = mva - Fv = (ma - F)v = 0$$

since $F = ma$. This means that the total energy never changes.

These are just a few of the examples of how derivatives come up in physics. In fact, most of physics, and especially electromagnetism and quantum mechanics, is governed by differential equations in several variables.

4.1 Elasticity of Demand

The elasticity of demand E, is the percentage rate of decrease of demand per percentage increase in price. We obtain it from the demand equation according to the following formula:$E = \frac{dq}{dp} \times \frac{p}{q}$

Where the demand equation expresses demand q, as a function of unit price p, we say that demand has unit elasticity if E=1.

To find the unit price that maximizes revenue, we express E as a function of p, set E-1, and then solve for p

EXAMPLE:

Suppose that the demand equation $q \ = \ 20{,}000 - 2p$.

Then $E = -(-2)\frac{p}{20000-2p} = \frac{p}{10000-p}$

$If\ p = \ 2000$, then $E = \frac{1}{4}$ and demand is inelasticity at this price.

If $p = \ 8000$, then $E \ = \ 4$, and demand is elasticity at this price.

If $p = \ 5000$, then $E \ = \ 1$, and the demand has unit elasticity at this price.

5. Application of Derivative in Mathematics:

Applications of Maxima and Minima: Optimization Problems:

We solve **optimization problems** of the following form: Find the values of the unknowns x, y,... maximizing (or minimizing) the value of the **objective function** f, subject to **certain constraints.** The constraints are equations and inequalities relating or restricting the variables x, y.....

To solve such a problem, we use the constraint equations to write all of the variables in terms of one chosen variable, substitute these into the objective function f, and then find extrema as above. (We use any constraint inequalities to determine the domain of the resulting function of one variable.) Specifically:

1. **Identify the unknown(s):**
   These are usually the quantities asked for in the problem.
2. **Identify the objective function.** This is the quantity you are asked to maximize or minimize.
3. **Identify the constraint(s).**
   These can be equations relating variables or inequalities expressing limitations on the values of variables.
4. **State the optimization problem.** This will have the form "Maximize [minimize] the objective function subject to the constraint(s)."
5. **Eliminate extra variables.**
   If the objective function depends on several variables, solve the constraint equations to express all variables in terms of one particular variable. Substitute these expressions into the objective function to rewrite it as a function of a single variable. Substitute the expressions into any inequality constraints to help determine the domain of the objective function.
6. **Find the absolute maximum (or minimum) of the objective function.**

Example:

Here is a maximization problem

Objective Function

Maximize $A = xy$

subject to $x + 2y = 100$

$x \geq 0$, and $y \geq 0$Constraints

Let us carry out the procedure for solving. Since we already have the problem stated as an optimization problem, we can start at Step 5.

5. Eliminate extra variables.

We can do this by solving the constraint equation $x + 2y = 100$ for x

($getting\ x = 100 - 2y$) and substituting in the objective function and the

Inequality involving x:

6. Find the Absolute maximum (or minimum) of the objective function:-

Now, we have to find the maximum value of $A = 100y - 2y^2$.

Taking derivative of A with respect to y.

$$\frac{dA}{dy} = \frac{d}{dy}(100y - 2y^2) = 100 - 4y$$

For extreme points

$$\frac{dA}{dy} = 0100 - 4y = 0$$

$$y = \frac{100}{4} y = 25$$

Put value of y in constant x, $x + 2y = 100$.

$$x = 100 - 2y$$

$$x = 100 - 2(25)$$

$x = 100 - 50$ x = 50

Thus, extreme point is (50,25).

Maximum value of objective function, $A = xyA = (50)(25)\ A = 1250$

Maximum, $A = 1250$

5.1 Analyzing Graphs:

We can use graphing technology to draw a graph, but we need to use differential calculus to understand what we are seeing. The most interesting features of a graph are the following.

Features of a Graph

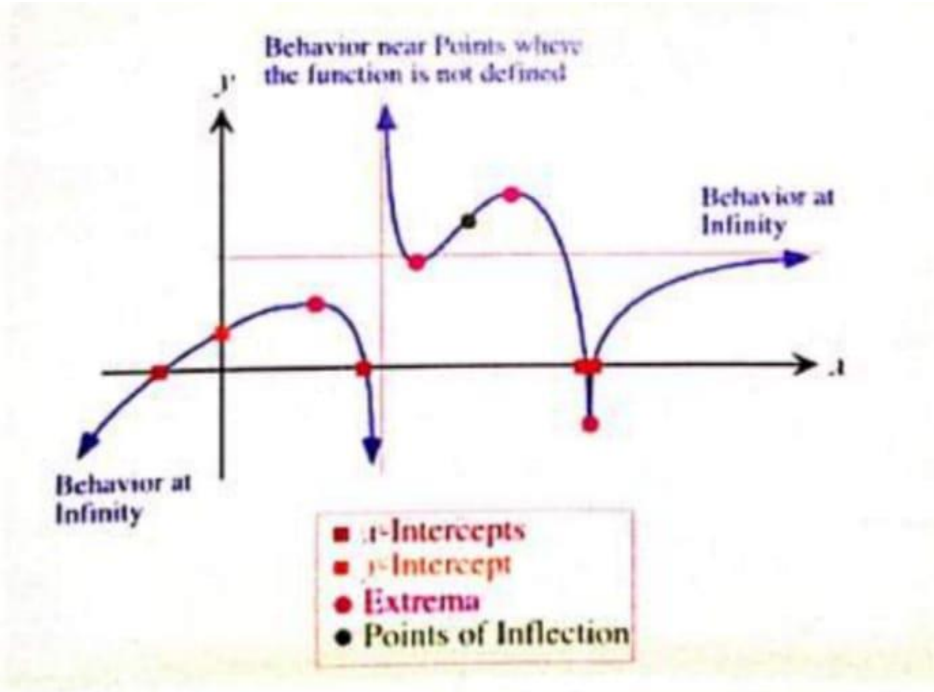
1. The x- and y-intercepts: If $y = f(x)$, find the x-intercept(s) by setting y = 0 and solving for x; find the y-intercept by setting x = 0.

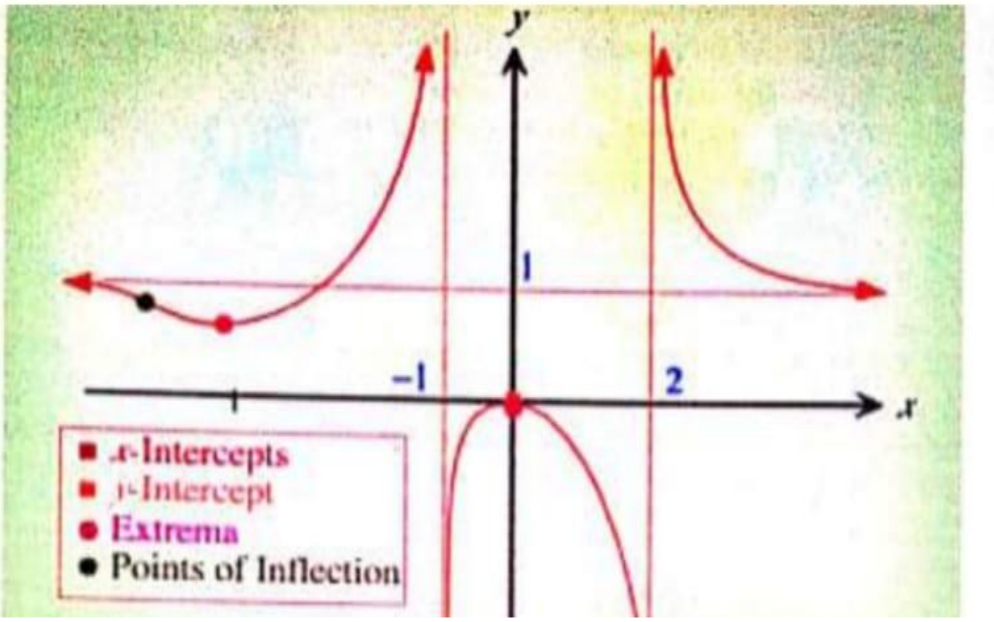
2. **Relative extrema**: Use the processer to find relative extrema and locate the relative extrema.

3. **Points of inflection**: Set $f"(x) = 0$ and solve for x to find candidate points of inflection.

4. **Behavior near points where the function is not defined:** If $f(x)$ is not defined at x, consider lim $f(x)$ and lim $f(x)$ to see how the graph of f approaches this point.

5. **Behavior at infinity:** Consider $lim\ f(x)$ and $lim\ f(x)$ if appropriate, +80 to see how the graph of f behaves far to the left and right.





To analyze this, we follow the procedure at left:

1. **The x and y-intercepts:** Setting y = 0 and solving for x gives $x = 0$. This is the only x-intercept. Setting $x = 0$ and solving for y gives y= 0: the y-intercept.

2. **Relative extrema:** The only extrema are stationary points found by setting $f'(x) = 0$ and solving for x, giving x = 0 and x=-4. The corresponding points on the graph are the relative maximum (0, 0) and the relative minimum at $(-4, 8/9)$.

3. **Points of inflection:** Solving f "(x) = 0 analytically is difficult, so we can solve it numerically (plot the second derivative and estimate where is crosses the x-axis) and find that the point of inflection lies at $x = -6.1072$.

4. **Behavior near points where the function is not defined:** The function is not defined at $x = -1$ and $x = 2$. the limits as x approaches these values from the inferred from the graph: left and right can be

Other Application of Derivatives in Mathematics:

- Approximation by differentials and newton's method
- Monotonic functions, relative and absolute extrema of functions
- Convex functions, inflection points and asymptotes
- Curve sketching
- L'Hospitals rule and indeterminate forms
- Roll's and mean value theorems
- Classical inequalities
- tangent, normal lines, curvature and radius of curvature
- Evaluate and involute
- Envelope of a family of curves and osculating curves
- related rates
- optimization problems in geometry, physics and economics

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