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Non Static Plane Symmetric Cosmic Strings Cosmological Model in *f(R)* Theory of Gravity

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Abstract:

In this paper, we investigate three non-statics plane symmetric cosmological models filled with cosmic strings in $f(R)$ theory of gravity. Three models are filled with three different cosmic strings. To solve the field equation, we used two conditions, the shear scalar σ is proportional to expansion scalar θ and the power law between the scalar function F and average scale factor a .We have found physical and Kinematical parameters of each model which are useful in study of cosmology. Also we obtained Ricci scalar of each model.

Keywords: Non Static Plane Symmetric Metric, *f(R)* Gravity, Cosmic String , Modified Gravity, Accelerating Model.

Introduction:

An accelerated expansion of the universe is the major problem in modern cosmology High-precision (Perlmutter et.al. [1], Riesset.al. [2],Astieret.al. [3], Miller et.al [4] &Bennette.L. [5]) indicates that universe is undergoing an accelerated expansion. The accelerating phase of cosmic expansion might be due to the pressure of mysterious energy and this energy composition of the universe has 4% ordinary matter,20% dark matter and 76% dark energy .The dark energy has large negative pressure while the pressure of dark matter is negligible .In order to explain the accelerated expansion of the universe, many authors proposed several modified theories of gravity. Significant among them are $f(R)$ theory of gravity where R is the scalar curvature, $f(T)$ theory, where T is the trace of the energy momentum tensor and $f(R,T)$ theory of gravity, where R is the Scalar curvature and T is the trace of energy momentum.

Recently, to investigate the DE model, many Researchers are attracted towards these modified theories of gravity. $f(R)$ theory of gravity in which the non-linear modifications of Einstein gravity is established [6, 7, 8, 9, 10, 11] and this $f(R)$ theory of gravity is considered to be the most suitable due to its cosmological importance .The model in this theory consist of higher order curvature invariant as functions of Ricci scalar R .Weyl [12] and Eddington [13] have discussed various action integrals of $f(R)$ gravity. Nojiri and odintsov [14, 15] have derived that a unification of the early time inflation and late time acceleration is allowed in $f(R)$ theory. Buchdalh [16] have studied different action principles in the constant of non singular oscillating cosmologies .Capozziallo et.al. [17] have discussed spherically symmetric solutions of $f(R)$ theory of gravity via Noether symmetry approach. Hollenstein and Lobo [18] have investigated the exact solutions of static spherically symmetric space-time in $f(R)$ gravity coupled to non linear electrodynamics. $f(R)$ Theory explained several feature [19, 20, 21] including singularity problem [22], gravitational stability [23], Newtonian limit [24] and solar system test [25].Sharif and Shamir [26], Sharif and Kausar[27] & Singh [28] discussed non-vacuum Bianchi type models in $f(R)$ theory of gravity. Shamir [29, 30], Reddy et. al. [31], Amir and Sattar [32] discussed exact vaccum solutions of Bianchi space times in $f(R)$ theory of gravity. Momeni and Gholizade have studied [33] cylindrically symmetric non-vacuum solution and Azadi et.al. [34] Have discussed cylindrically symmetric non-vacuum solution. Y.aditya et.al. [35] Studied the non-vacuum plane symmetric universe in $f(R)$ gravity.

Letelier [36] initiated the treatment of string and he has obtained solution to Einstein field equation for a cloud of the strings with spherical, plane and cylindrical symmetry. Later [37] by solving the Einstein field equation for a cloud massive strings, he obtained Bianchi type-I cosmological model and kantawski-sach space time. Also Wang [38] explored string cosmological models in kantawski-sach space time. Anirudhpradhan et.al. [39] Discussed Bianchi type –I massive string cosmological model in general relativity. In which they assumed the law of variation of scalar factor as increasing function of time. R. vankteswara et.al [40] discussed LRS Bianchi type-I $f(R)$ in theory of gravity in inflationary string cosmological model in Brans-Dicke theory of gravitation.Also same author [41] studied the LRS Bianchi type-III massive string cosmological model in scalar theory of gravitation .Tikekar and patel [42] obtained some exact Bianchi type-III cosmological solution of massive strings in presence and absence of magnetic field. An axially symmetric Bianchi type-I string dust cosmological model in presence and absence of magnetic field is investigated by Banerjee [43].Chakraborty [44] have discussed string cosmological model with magnetic field. Bali et.al.[45-46] discussed Bianchi type-I and IX string cosmological models in general relativity.Adhav K.S.[47] studied Bianchi type-III cosmological model.After noting such development in $f(R)$ gravity, we aim to study different cosmological models in $f(R)$ theory of gravity with cosmic strings .Using three cosmic string here we discussed the non-static plane symmetric cosmological models in $f(R)$ theory of gravity. Also we obtained the physical and kinematical parameters and corresponding Ricci scalar for these models. the paper is organized as below in section $f(R)$ theory of gravity, metric and field equations ,type of strings, model with Geometrical strings ,model with Reddy strings, and model with Takabayasi string in last section summary and conclusion.

f(R) Theory of Gravity

Field equation in *f(R)* theory of gravity are obtained from the action

$$S=\int\left(\frac{1}{2k}\sqrt{-g}\,f(R)+L_m\sqrt{-g}\right)d^4x \qquad (1)$$

Here $f(R)$ is a general function of the Ricci scalar, where $k=8\pi G$, g is a determinant of metric $g_{\mu\nu}$ and L_m is metric Lagrangian that depend on a $g_{\mu\nu}$.

It is noted that this action is obtain just by replacing R by $f(R)$ in standard Einstein-Hilbert action.

The corresponding field equation are found by varying the action with respect to the metric $g_{\mu\nu}$

$$F(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\Box F(R)=kT_{\mu\nu} \qquad (2)$$

Where $F(R) = \dfrac{df(R)}{dR}$, $\Box = \nabla^{\mu}\nabla_{\nu}$ (3)

∇_{ν} is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m .

After contraction the field equation we get

$$F(R)R - 2f(R) + 3\Box\ F(R) = kT \quad (4)$$

Using this value of f(R) non-vacuum field equation (2), we have

$$\frac{F(R)R - \nabla_{\mu}\nabla_{\nu} F(R) - kT_{\mu\nu}}{g_{\mu\nu}} = \frac{1}{4}[F(R) - \Box F(R) - kT] \quad (5)$$

In the above equation, the terms on the right hand side are independent of index μ, so we can write the field equation in the Following manner,

$$A_{\mu\nu} = \frac{F(R)R_{\mu\nu} - \nabla_{\mu}\nabla_{\nu}F(R) - kT_{\mu\nu}}{g_{\mu\nu}} \quad (6)$$

In Equation (4) is an important relationship between $f(R)$ and $F(R)$ which will be used to simplify the field equation and to evaluate $f(R)$. we consider energy momentum tensor for the source of cosmic strings is given by

$$T_{ij} = \rho\mu_i\mu_j - \lambda x_i x_j \quad \text{and} \quad \rho = \rho_p + \lambda \quad (7)$$

Where ρ is the rest energy density of strings, λ is the tension density of strings, μ_i is four velocities and x_i is the direction of anisotropy of string. ρ_p is the rest energy density of the particles attach to the string.

$$\mu^i\mu_j = -x^i x_j = 1 \quad , \quad \mu^i x_i = 0 \quad (8)$$

The axis of symmetry of cosmic string source is along z-axis.

In the co-moving co-ordinate system, from equation (7)we have

$$T_1^1 = T_2^2 = 0 \quad , \quad T_3^3 = \lambda \quad , \quad T_4^4 = \rho \ , \ T_j^i = 0 \ \ for\ i \neq j \quad (9)$$

The quantities ρ and λ depend on t only

Metric and Field equations

Thenon static plane symmetric model characterized by spatial homogeneity and anisotropy is defined by the metric in the form

$$ds^2 = e^{2h}\left(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2\right) \quad (10)$$

Where h and S are function of time t only

Ricci scalaris

$$R = 2e^{-2h}\left[3\ddot{h} + 3\dot{h}^2 + 3\frac{\dot{h}\dot{S}}{S} + \frac{\ddot{S}}{S}\right] \tag{11}$$

Here overhead dot denotes the derivative with respect to cosmic time t

Using equation (6) (7) and (9), the corresponding field equation (2) we get the following set of equations

$$A_1 = A_2 = e^{-2h}\left[\left(\ddot{h} + 2\dot{h}^2 + \frac{\dot{h}\dot{S}}{S}\right)F + \dot{h}\dot{F}\right]$$

$$A_3 = e^{-2h}\left[\left(\frac{\ddot{S}}{S} + \ddot{h} + 2\dot{h}^2 + \frac{3\dot{h}\dot{S}}{S}\right)F + \left(\dot{h} + \frac{\dot{S}}{S}\right)\dot{F}\right] - k\lambda$$

$$A_4 = e^{-2h}\left[\left(3\ddot{h} + \frac{\ddot{S}}{S} + \frac{\dot{h}\dot{S}}{S}\right)F + \ddot{F}\right] - k\rho$$

These equations are independent of μ,

By $A_2 - A_4 = 0$ and $A_3 - A_4 = 0$, we get

$$e^{-2h}\left[-2\ddot{h} + \frac{2\dot{h}\dot{S}}{S} + 2\dot{h}^2 + \frac{\dot{S}\dot{F}}{SF} + \frac{\dot{h}\dot{F}}{F} + \frac{\ddot{F}}{F}\right] = \frac{(\lambda - \rho)k}{F} \tag{12}$$

$$e^{-2h}\left[-2\ddot{h} + 2\dot{h}^2 - \frac{\ddot{S}}{S} + \frac{\dot{h}\dot{F}}{F} + \frac{\ddot{F}}{F}\right] = \frac{-\rho k}{F} \tag{13}$$

The spatial volume is given by

$$V = S\,e^{4h} \tag{14}$$

The average scale factor is

$$a = \left(S\,e^{4h}\right)^{\frac{1}{3}} \tag{15}$$

Using equation (21), the average Hubble's parameter is defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{1}{3}\left(2\dot{h} + 2\dot{h} + \frac{\dot{S}}{S}\right) = \frac{1}{3}\left(4\dot{h} + \frac{\dot{S}}{S}\right) \tag{16}$$

Where $H_1 = H_2 = 2\dot{h}$ $\;, H_3 = \frac{\dot{S}}{S}$ are the directional Hubble's parameters in the directions of $r, \theta\, and\, z$ respectively.

The expansion scalar (θ) and shear scalar (σ^2) are given by $\theta = 3H = \left(4\dot{h} + \frac{\dot{S}}{S}\right)$ (17)

and $\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right]$ (18)

The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \quad (19)$$

Average anisotropic parameter is given by

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{3}\frac{\sigma^2}{H^2} \quad (20)$$

Where H_i for i =1, 2, 3 are directional Hubble's parameters.

Type of String:

The simplest proportionality relation between ρ and λ is given by

$$\rho = \alpha\lambda \quad (21)$$

This relation gives three different types of String

I}Geometric string or Nambu string

$$\alpha = 1 \quad , \rho = \lambda \quad (22)$$

II}Reddy string

$$\alpha = -1 \ , \ \rho = -\lambda \quad (23)$$

III}Takabayasi stringor p- string

$$\alpha = 1 + \omega \ , \omega \geq 0 \qquad \rho = (1+\omega)\lambda \quad (24)$$

Corresponding to these three different types of strings, three different models of the universe are obtained.

1) **Model filled with Geometric String or Nambu String**

The filled equations (12) & (13) are a system of two independent equations with five unknowns (S, h, F, λ and ρ). hence to find the solution. we use the following two conditions

i) The shear scalar σ is proportional to expansion scalar θ, so that

we get $e^h = S^k$, Where $k \neq 1$ is a constant. (25)

and**ii)**Power law relation between the function F and the average scale factor given by

$$F(R) = F_0\left[a(t)\right]^m \tag{26}$$

Where m is an arbitrary constant and F_0 is proportionality constant.

Use equation (22) in field equation (12) and (13)

$$e^{-2h}\left[-2\ddot{h} + \frac{2\dot{h}\dot{S}}{S} + 2\dot{h}^2 + \frac{\dot{S}\dot{F}}{SF} + \frac{\dot{h}\dot{F}}{F} + \frac{\ddot{F}}{F}\right] = 0$$

$$-2\ddot{h} + \frac{2\dot{h}\dot{S}}{S} + 2\dot{h}^2 + \frac{\dot{S}\dot{F}}{SF} + \frac{\dot{h}\dot{F}}{F} + \frac{\ddot{F}}{F} = 0 \tag{27}$$

$$e^{-2h}\left[2\ddot{h} - 2\dot{h}^2 + \frac{\ddot{S}}{S} - \frac{\dot{h}\dot{F}}{F} - \frac{\ddot{F}}{F}\right] = \frac{k\rho}{F} = \frac{k\lambda}{F} \tag{28}$$

Using equation (25) and (26) in equation (27), we get

$$\frac{\ddot{S}}{S} + \frac{8}{3}k^2(m-1)\frac{\dot{S}^2}{S^2} = 0$$

$$\frac{\ddot{S}}{\dot{S}} + \beta\frac{\dot{S}}{S} = 0 \tag{29}$$

Where $\beta = \frac{8}{3}k^2(m-1)$

Its solution is

$$S = \left[(c_1 t + c_2)(\beta+1)\right]^{\frac{1}{\beta+1}}$$

Where c_1 and c_2 are constant of integration.

$$S = \phi\left[(c_1 t + c_2)\right]^{\frac{1}{\beta+1}} \tag{30}$$

Where $\phi = (\beta+1)^{\frac{1}{\beta+1}}$

Using equation (25) and (30) in metric (10), we get

$$ds^2 = S^{2k}\left(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2\right)$$

$$ds^2 = \phi^{2k}\left[(c_1 t + c_2)\right]^{\frac{2k}{\beta+1}}\left(dt^2 - dr^2 - r^2 d\theta^2 - \phi^2\left[(c_1 t + c_2)\right]^{\frac{2}{\beta+1}} dz^2\right) \quad (31)$$

With proper choice of co-ordinate $c_1 t + c_2 = T$

From equation (30)

$$S = \phi\left[(T)\right]^{\frac{1}{\beta+1}} \quad (32)$$

From equation (31) and (32), we get

$$ds^2 = \phi^{2k} T^{\frac{2k}{1+\beta}}\left[\frac{dT^2}{c_1^2} - dr^2 - r^2 d\theta^2 - \phi^2 T^{\frac{2}{1+\beta}} dz^2\right] \quad (33)$$

Physical and Kinematical parameter:

The metric (33) together with the following physical parameters constitutes a non-static plane symmetric universe in f (R) theory of gravity.

From Eqs. (14), the spatial volume of the model is

$$V = S\, S^{4k} = S^{(1+4k)} = \phi^{(1+4k)} T^{\frac{(1+4k)}{1+\beta}} \quad (34)$$

From Eqs. (15), the average scale factor is

$$a = \left(S\, e^{2h}\right)^{\frac{1}{3}} = \phi^{\frac{(1+4k)}{3}} T^{\frac{(1+4k)}{3(1+\beta)}} \quad (35)$$

From equation (16), the mean Hubble's parameters is given by

$$H = \frac{1}{3}\frac{(1+4k)c_1}{(1+\beta)T} \quad (36)$$

From equation (17), the expansion scalar, we get

$$\theta = \frac{(1+4k)c_1}{(1+\beta)T} \quad (37)$$

From equation (18) we obtained, shear scalar (σ^2) are given by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right] = \frac{(2k-1)^2}{3}\frac{c_1^2}{(1+\beta)^2 T^2} \quad (38)$$

We get, the deceleration parameter q is

$$q=\frac{(3\beta-4k+2)}{(1+4k)} \tag{39}$$

Average anisotropic parameter is given by

$$\overline{A}=\frac{2(4k^2-4k+1)}{16k^2+8k+1} \tag{40}$$

From equation (26), the function $F(R)$ is given by

$$F=F_0\,\phi^{\frac{(1+4k)m}{3}}\,T^{\frac{(1+4k)m}{3(1+\beta)}} \tag{41}$$

Using equation (13), energy density of the model is obtained as

$$\rho=\lambda=\frac{F_0c_1^2\phi^{\frac{m(1+4k)-6k}{3}}}{k(1+\beta)^2}T^{\frac{m(1+4k)-6(1+k+\beta)}{3(1+\beta)}}\left[\begin{pmatrix}\frac{mk}{3}\\+\frac{m}{3}\\-2k-1\end{pmatrix}\beta-\begin{pmatrix}2k+2k^2-\frac{7}{9}mk\\+\frac{m(m-1)(1+4k)^2}{9}\\+\frac{4mk^2}{9}-\frac{2m}{9}\end{pmatrix}\right] \tag{42}$$

Using equation (11), Corresponding Ricci scalar is

$$R=\frac{2c_1^2\phi^{-2k}}{(1+\beta)^2}T^{\frac{-2(1+k+\beta)}{(1+\beta)}}\left[3k^2-\beta(1+3k)\right] \tag{43}$$

From Eqs. (4), (41)–(43), we get the function f(R) as

$$f(R)=\frac{F_0c_1^2}{2(1+\beta)^2}\phi^{\frac{m(1+4k)-6k}{3}}T^{\frac{m(1+4k)-6(1+k+\beta)}{3(1+\beta)}}\begin{bmatrix}\left(\frac{1}{3}mk+\frac{1}{3}m+1\right)\beta-2k^2\\+m(1+3k)(1+4k)\\-\frac{20}{9}mk-\frac{4}{9}mk^2+\frac{2}{9}m\\-\frac{1}{9}m(m-1)(1+4k)^2\end{bmatrix} \tag{44}$$

II.Model filled with Reddy String

Using equation (23) in equation (12),(13), we get

$$e^{-2h}\left[2\ddot{h}-\frac{2\dot{h}\dot{S}}{S}-2\dot{h}^2-\frac{\dot{S}\dot{F}}{SF}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}\right]=\frac{-2k\lambda}{F} \tag{45}$$

$$e^{-2h}\left[2\ddot{h}-2\dot{h}^{2}+\frac{\ddot{S}}{S}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}\right]=\frac{k\rho}{F}=\frac{-k\lambda}{F} \tag{46}$$

Using equation (45) and (46), we get

$$e^{-2h}\left[-2\ddot{h}-\frac{2\dot{h}\dot{S}}{S}-\frac{2\ddot{S}}{S}+2\dot{h}^{2}-\frac{\dot{S}\dot{F}}{SF}+\frac{\dot{h}\dot{F}}{F}+\frac{\ddot{F}}{F}\right]=0$$

$$2\ddot{h}+\frac{2\dot{h}\dot{S}}{S}+\frac{2\ddot{S}}{S}-2\dot{h}^{2}+\frac{\dot{S}\dot{F}}{SF}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}=0 \tag{47}$$

$$(1+k)\frac{\ddot{S}}{S}+\frac{8}{3}k^{2}(m-1)\frac{\dot{S}^{2}}{S^{2}}=0$$

$$\frac{\ddot{S}}{\dot{S}}+\frac{\beta}{1+k}\frac{\dot{S}}{S}=0 \tag{48}$$

Where $\beta=\frac{8}{3}k^{2}(m-1)$

Its solution is

$$S=\left[(c_{3}t+c_{4})\left(\frac{1+\beta+k}{1+k}\right)\right]^{\frac{1+k}{\beta+1+k}},$$

Where c_3 and c_4 are constant of integration.

$$S=\psi\left[(c_{3}t+c_{4})\right]^{\frac{1+k}{1+\beta+k}} \tag{49}$$

Where $\psi=\left(\frac{1+\beta+k}{1+k}\right)^{\frac{1+k}{\beta+1+k}}$

With proper choice of co-ordinate $c_3 t+c_4=T$

From equation (49)

$$S=\psi\left[(T)\right]^{\frac{1+k}{\beta+1+k}} \tag{50}$$

Using equation (25) and (49) in metric (10), we get

$$ds^{2}=S^{2k}\left(dt^{2}-dr^{2}-r^{2}d\theta^{2}-S^{2}dz^{2}\right)$$

$$ds^{2}=\psi^{2k}\left[(c_{3}t+c_{4})\right]^{\frac{2k(1+k)}{\beta+1+k}}\left(dt^{2}-dr^{2}-r^{2}d\theta^{2}-\psi^{2}\left[(c_{3}t+c_{4})\right]^{\frac{2(1+k)}{\beta+1+k}}dz^{2}\right) \tag{51}$$

Using Eqns (50), we get

$$ds^2 = \psi^{2k} T^{\frac{2k(1+k)}{1+k+\beta}} \left[\frac{dT^2}{c_3^2} - dr^2 - r^2 d\theta^2 - \psi^2 T^{\frac{2(1+k)}{1+k+\beta}} dz^2 \right] \tag{52}$$

Physical and Kinematical parameter:

From Eqs. (14), The spatial volume of the model is

$$V = S\,S^{4k} = S^{(1+4k)} = \psi^{(1+4k)} T^{\frac{(1+4k)(1+k)}{1+\beta+k}} \tag{53}$$

From Eqs. (15), we define as the average scale factor

$$a = \left(S\,e^{2h}\right)^{\frac{1}{3}} = \psi^{\frac{(1+4k)}{3}} T^{\frac{(1+4k)(1+k)}{3(1+\beta+k)}} \tag{54}$$

From equation (16), the mean Hubble's parameters is given by

$$H = \frac{1}{3} \frac{(1+4k)(1+k)\,c_3}{(1+\beta+k)T} \tag{55}$$

From equation (17), the expansion scalar, we get

$$\theta = \frac{(1+4k)(1+k)\,c_3}{(1+\beta+k)T} \tag{56}$$

From equation (18) we obtained, shear scalar (σ^2) aregiven by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right] = \frac{c_3^2 (1+k)^2 (2k-1)^2}{3T^2 (1+k+\beta)^2} \tag{57}$$

We get, the deceleration parameter q is given by

$$q = \frac{\left(3\beta - 2k - 4k^2 + 2\right)}{(1+4k)(1+k)} \tag{58}$$

Average anisotropic parameter is given by

$$\bar{A} = \frac{2\left(4k^2 - 4k + 1\right)}{16k^2 + 8k + 1} \tag{59}$$

From equation (26), the function $F(R)$ is given by

$$F = F_0\, \psi^{\frac{(1+4k)m}{3}} T^{\frac{(1+4k)(1+k)m}{3(1+\beta+k)}} \tag{60}$$

The energy density in the model is obtained, in terms of the Reddy string $\rho = -\lambda$.

Hence from Eqns. (13) we obtain

$$\lambda = \frac{F_0 c_3^2 (1+k)\psi^{\frac{m(1+4k)-6k}{3}}}{k(1+k+\beta)^2} T^{\frac{m(1+4k)(1+k)-6\left(k^2+1+2k+\beta\right)}{3(1+\beta+k)}} \left[\left(\frac{mk}{3} + \frac{m}{3} - 2k - 1 \right)\beta - \left(2k+2k^2 - \frac{7}{9}mk + \frac{m(m-1)(1+4k)^2}{9} + \frac{4mk^2}{9} - \frac{2m}{9} \right)(1+k) \right]$$

$$\rho = \frac{-F_0 c_3^2 (1+k)\psi^{\frac{m(1+4k)-6k}{3}}}{k(1+k+\beta)^2} T^{\frac{m(1+4k)(1+k)-6\left(k^2+1+2k+\beta\right)}{3(1+\beta+k)}} \left[\left(\frac{mk}{3} + \frac{m}{3} - 2k - 1 \right)\beta - \left(2k+2k^2 - \frac{7}{9}mk + \frac{m(m-1)(1+4k)^2}{9} + \frac{4mk^2}{9} - \frac{2m}{9} \right)(1+k) \right] \quad (61)$$

From equation (11), Corresponding Ricci scalar is

$$R = \frac{2c_3^2 \psi^{-2k}}{(1+k+\beta)^2} T^{\frac{-2\left(k^2+1+2k+\beta\right)}{(1+\beta+k)}} \left[3k^2(1+k)^2 - \beta(1+3k)(1+k) \right] \quad (62)$$

From Eqs. (4), (60) and (62), we get the function *f(R)* as

$$f(R) = \frac{F_0 c_3^2}{2(1+\beta+k)^2} \psi^{\frac{m(1+4k)-6k}{3}} T^{\frac{m(1+4k)(1+k)-6\left(k^2+1+2k+\beta\right)}{3(1+\beta+k)}} \left[\left(mk + m - 6k - 1 \right)\beta - \left(6k^2 + m(1+3k)(1+4k) + \frac{2}{3}mk + \frac{4}{3}mk^2 - \frac{2}{3}m - \frac{1}{3}m(m-1)(1+4k)^2 \right)(1+k) \right] \quad (63)$$

III. Model filled with Takabayasistring

Using equation (24) in equation (12), (13)

$$e^{-2h}\left[2\ddot{h}-\frac{2\dot{h}\dot{S}}{S}-2\dot{h}^{2}-\frac{\dot{S}\dot{F}}{SF}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}\right]=\frac{-\omega k\lambda}{F} \quad (64)$$

$$e^{-2h}\left[2\ddot{h}-2\dot{h}^{2}+\frac{\ddot{S}}{S}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}\right]=\frac{k\rho}{F}=\frac{k(1+\omega)\lambda}{F} \quad (65)$$

$$e^{-2k}\left(\frac{-2\dot{h}\dot{S}}{S}-\frac{\ddot{S}}{S}-\frac{\dot{S}\dot{F}}{SF}\right)=\frac{-k(1+\omega)\lambda}{F} \quad (66)$$

Adding Eqns (65) and Eqns (66), we get

$$e^{-2h}\left[2\ddot{h}-\frac{2\dot{h}\dot{S}}{S}-2\dot{h}^{2}-\frac{\dot{S}\dot{F}}{SF}-\frac{\dot{h}\dot{F}}{F}-\frac{\ddot{F}}{F}\right]=0$$

$$k\frac{\ddot{S}}{S}+\frac{8}{3}k^{2}(m-1)\frac{\dot{S}^{2}}{S^{2}}=0$$

$$\frac{\ddot{S}}{\dot{S}}+\frac{\beta}{k}\frac{\dot{S}}{S}=0 \quad (67)$$

Where $\beta=\frac{8}{3}k^{2}(m-1)$

Its solution is $S=\left[(c_5t+c_6)\left(\frac{\beta+k}{k}\right)\right]^{\frac{k}{\beta+k}}$

Where c_5 and c_6 are constant of integration.

$$S=\varphi\left[(c_5t+c_6)\right]^{\frac{k}{\beta+k}} \quad (68)$$

Where $\varphi=\left(\frac{\beta+k}{k}\right)^{\frac{k}{\beta+k}}$

With proper choice of co-ordinate $c_5t+c_6=T$

From equation (68)

$$S=\varphi\left[(T)\right]^{\frac{k}{\beta+k}} \quad (69)$$

Using equation (25) and (68) in metric (10), we get

$$ds^2 = S^{2k}\left(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2\right)$$

$$ds^2 = \varphi^{2k}\left(c_5 t + c_6\right)^{\frac{2k^2}{k+\beta}}\left[dt^2 - dr^2 - r^2 d\theta^2 - \varphi^2\left(c_5 t + c_6\right)^{\frac{2k}{k+\beta}} dz^2\right] \tag{70}$$

From equation (69)

$$ds^2 = \varphi^{2k}(T)^{\frac{2k^2}{\beta+k}}\left(\frac{dT^2}{c_5^2} - dr^2 - r^2 d\theta^2 - \varphi^2 [T]^{\frac{2k}{\beta+k}} dz^2\right) \tag{71}$$

Physical and Kinematical parameter:

From Eqs. (14), the spatial volume of the model is

$$V = S\,S^{4k} = S^{(1+4k)} = \varphi^{(1+4k)} T^{\frac{(1+4k)k}{\beta+k}} \tag{72}$$

From Eqs. (15), we define as the average scale factor

$$a = \left(S\,e^{2h}\right)^{\frac{1}{3}} = \varphi^{\frac{(1+4k)}{3}} T^{\frac{(1+4k)k}{3(\beta+k)}} \tag{73}$$

From equation (16), the mean Hubble's parameters is given by

$$H = \frac{1}{3}\frac{(1+4k)k\,c_5}{(\beta+k)T} \tag{74}$$

From equation (17), the expansion scalar, we get

$$\theta = \frac{(1+4k)k\,c_5}{(\beta+k)T} \tag{75}$$

From equation (18) we obtained, shear scalar (σ^2) aregiven by

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right] = \frac{k^2 c_5^2 (2k-1)^2}{3T^2 (k+\beta)^2} \tag{76}$$

We get, the deceleration parameter q is given by

$$q = \frac{\left(3\beta - 4k^2 + 2k\right)}{(1+4k)k} \tag{77}$$

Average anisotropic parameter is given by

$$\bar{A} = \frac{2\left(4k^2 - 4k + 1\right)}{\left(16k^2 + 8k + 1\right)} \tag{78}$$

From equation (26), the function $F(R)$ is given by

$$F(R) = F_0\, \varphi^{\frac{(1+4k)m}{3}}\, T^{\frac{k(1+4k)m}{3(\beta+k)}} \tag{79}$$

The energy density in the model is obtained, in terms of the Reddy string $\rho = (1+\omega)\lambda$. Hence from Eqns.(13) we obtain

$$\lambda = \frac{F_0 c_5^2 \varphi^{\frac{m(1+4k)-6k}{3}}}{(1+\omega)(k+\beta)^2} T^{\frac{m(1+4k)k-6(k^2+k+\beta)}{3(\beta+k)}} \left[\begin{pmatrix} \frac{mk}{3} \\ +\frac{m}{3} \\ -2k \\ -1 \end{pmatrix} \beta - \begin{pmatrix} 2k+2k^2 \\ -\frac{7}{9}mk \\ +\frac{m(m-1)(1+4k)^2}{9} \\ +\frac{4mk^2}{9} \\ -\frac{2m}{9} \end{pmatrix} (k) \right]$$

$$\rho = \frac{F_0 c_5^2\, \varphi^{\frac{m(1+4k)-6k}{3}}}{(k+\beta)^2} T^{\frac{m(1+4k)k-6(k^2+k+\beta)}{3(\beta+k)}} \left[\begin{pmatrix} \frac{mk}{3} \\ +\frac{m}{3} \\ -2k \\ -1 \end{pmatrix} \beta - \begin{pmatrix} 2k+2k^2 \\ -\frac{7}{9}mk \\ +\frac{m(m-1)(1+4k)^2}{9} \\ +\frac{4mk^2}{9} \\ -\frac{2m}{9} \end{pmatrix} (k) \right] \tag{80}$$

From equation (11), Corresponding Ricci scalar is

$$R = \frac{2c_5^2\, \varphi^{-2k}}{(k+\beta)^2} T^{\frac{-2(k^2+k+\beta)}{(\beta+k)}} \left[3k^4 - \beta(1+3k)k\right] \tag{81}$$

From Eqs. (4), (79) and (81), we get the function *f(R)* as

$$f(R)=\frac{kF_0 c_5^2}{2(\beta+k)^2}\varphi^{\frac{m(1+4k)-6k}{3}}T^{\frac{m(1+4k)k-6\left(k^2+k+\beta\right)}{3(\beta+k)}}\left[\begin{pmatrix} mk \\ +m \\ -6k \\ -1 \end{pmatrix}\beta-\begin{pmatrix} 6k^2 \\ +m(1+3k) \\ (1+4k) \\ +\frac{2}{3}mk \\ +\frac{4}{3}mk^2 \\ -\frac{2}{3}m \\ -\frac{1}{3}m(m-1) \\ (1+4k)^2 \end{pmatrix}k\right]-\frac{k(2+\omega)}{2}\lambda \qquad (82)$$

Summary and conclusion

An accelerated expansion of the universe has been considered as the major issue in the modern cosmology .In study of accelerated expansion of the universe modified theory of gravity is the best approach. Amongst modified theory of gravity, $f(R)$ theory of gravity, whose action is a non-linear function of the curvature scalar, R described the evolution of the universe. Here we have studies three cosmological models filled with cosmic strings in framework of $f(R)$ gravity theory. we have found the cosmological terms spatial volume, average scale factor, mean Hubble parameter ,expansion scalar, shear scalar, average anisotropic parameter for each model our conclusions are as below.

i) From equations of three models (33), (52), and (71) it is observed that the volume of the universe is zero at initial epoch and it increases with cosmic time. Thus the model represents the accelerated expansion of the universe.

ii) From the expression of mean Hubble parameter and expansion scalar of all these three models, it is observe that as initial stage expansion rate is more and it slow down with increase of time. iii) In each model, the ratio σ/θ indicates that the universe does not attain isotropy. iv) The models arehave initial singularity.

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